Block-selective algebraic multigrid in `foam-extend`

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The most powerful class of iterative linear solvers are the multigrid methods, *Trottenberg et al., Multigrid (2001)*. They were originally created for systems of discretised elliptic PDEs but were later expanded and have proven to be efficient for general types of PDEs. Multigrid methods exploit the fact that the point-fixed methods (e.g. Gauss–Seidel) tend to quickly reduce the high frequency solution errors, i.e. the errors whose direction corresponds to the largest eigenvalues of the matrix. However, the low frequency errors remain and this is why the performance (convergence) of the fixed-point methods deteriorates. To solve this issue, multigrid methods construct a hierarchy of grids by coarsening the initial grid. The low frequency errors on the fine grid become high frequency errors on the coarse grid and the fixed-point algorithms are able to efficiently reduce these errors. The correction obtained on the coarse grid is then transferred back to the fine grid. This procedure of fine-to-coarse and coarse-to-fine grid communication can be repeated multiple times in various directions which will be defined by the multigrid cycle type. Algebraic multigrid methods operate on matrix coefficients directly and do not need a computational grid. We present a newly implemented algebraic multigrid method for scalar and block matrices in `foam-extend`. The method is based on the classical multigrid coarsening algorithm (SAMG), *Stüben (2001)*. Construction of the coarse level matrix relies on choosing the representative equations according to the strength of connectivity in the fine level matrix. Interpolation for coarse-to-fine communication must be explicitly calculated by exploiting the behaviour of the algebraically smooth error:

$$a_{ii}e_i + \sum_{j\in\mathbb{N}_i} a_{ij}e_j = 0. \qquad (1)$$

$$a_{ii}e_i = -\sum_{j\in\mathbb{C}_i} a_{ij}e_j - \sum_{j\in\mathbb{F}_i^s} a_{ij}e_j - \sum_{j\in\mathbb{F}_i^w} a_{ij}e_j. \qquad (2)$$

$\mathbb{N}_i$ is the set of all neighbours of i,

$\mathbb{C}_i$ is the subset of $\mathbb{N}_i$ containing strong coarse neighbours of i,

$\mathbb{F}_i^w$ is the subset of of $\mathbb{N}_i$ containing strong fine neighbours of i,

$\mathbb{F}_i^s$ is the subset of of $\mathbb{N}_i$ containing weak fine neighbours of i.

The beforementioned subsets are formed by calculating the norm $|a_{ij}|$ of all block–matrix coefficients and forming a *primary matrix*. The first option is to simply use one of the coupled equations and calculate the interpolation based on its connectivity, without taking into account the variable cross–couplings (e.g. the pressure equation). Another option is to calculate a norm taking into account all equations and excluding the cross couplings (e.g. the magnitude of the block–coefficient diagonal).
The approximation of equation 1 yields interpolation weights for contribution from a single coarse equation to the fine equation:

$$w = -\alpha \frac{1}{|a_{ii}| + \sum_{j\in\mathbb{F}_i^w} |a_{ij}|} |a_{ij}^C|, \qquad (3)$$

where α is the scaling factor taking into account strong connections which are not coarse:

$$\alpha = \frac{\sum_{j\in\mathbb{C}_i} |a_{ij}| + \sum_{j\in\mathbb{F}_i^s} |a_{ij}|}{\sum_{j\in\mathbb{C}_i} |a_{ij}|} > 1. \qquad (4)$$

These weights are used for prolongation of the solution correction from coarse to fine level. The restriction matrix for transferring the residual from fine to coarse level is obtained as a transpose of the prolongation matrix. The coarse level matrix is formed by the triple product of restriction, fine level matrix and prolongation:

$$\mathbf{A}_{coarse} = \mathbf{R} \cdot \mathbf{A}_{fine} \cdot \mathbf{P}. \qquad (5)$$

The case chosen for showing the performance of block–SAMG is the cooling of an engine block. The first and second level of coarsening are shown in Fig. 1. Rainbow–coloured cells are chosen as coarse and solved on the next level.

Figure 1: First and second coarsening level using the pressure equation as the primary matrix

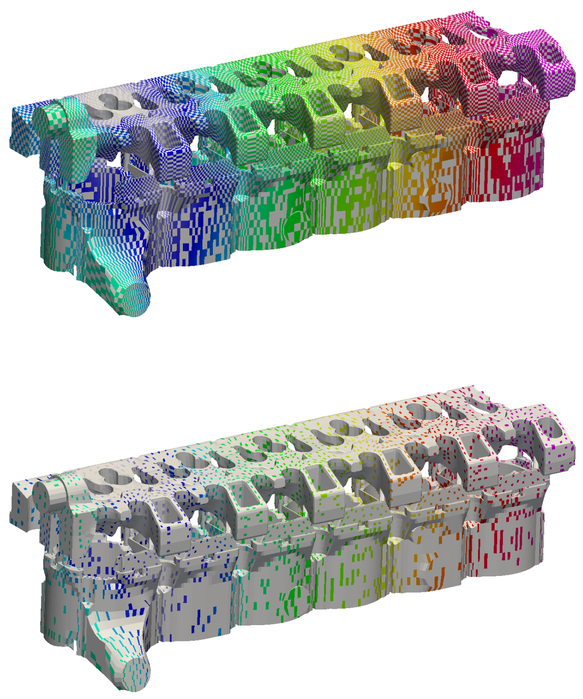

Figure 2: Comparison of SAMG using pressure-equation and two norm for the primary matrix, BiCGStab and AAMG

	1.	2.	3.	4.
Algorithm	SAMG	SAMG	BiCGStab	AAMG
Norm	Pressure	Two-norm	/	Pressure
Relative tolerance	0	0.001	0	0.001
Converged	250 s	6000 s	600 s	2500 s
Number of linear iterations	229	11000	6709	12000

Convergence of SAMG is compared to convergence of BiCGStab, a robust Krylov subspace solver, *Saad (2000)*, and aggregative AMG (AAMG), Tab. 2. The pressure equation, excluding the pressure–velocity cross–couplings, was used as the criterion for coarsening. It converges faster than the case with coarsening using two–norm of the block coefficients.

Figure 3: Convergence of SAMG in comparison with BiCGStab and AAMG

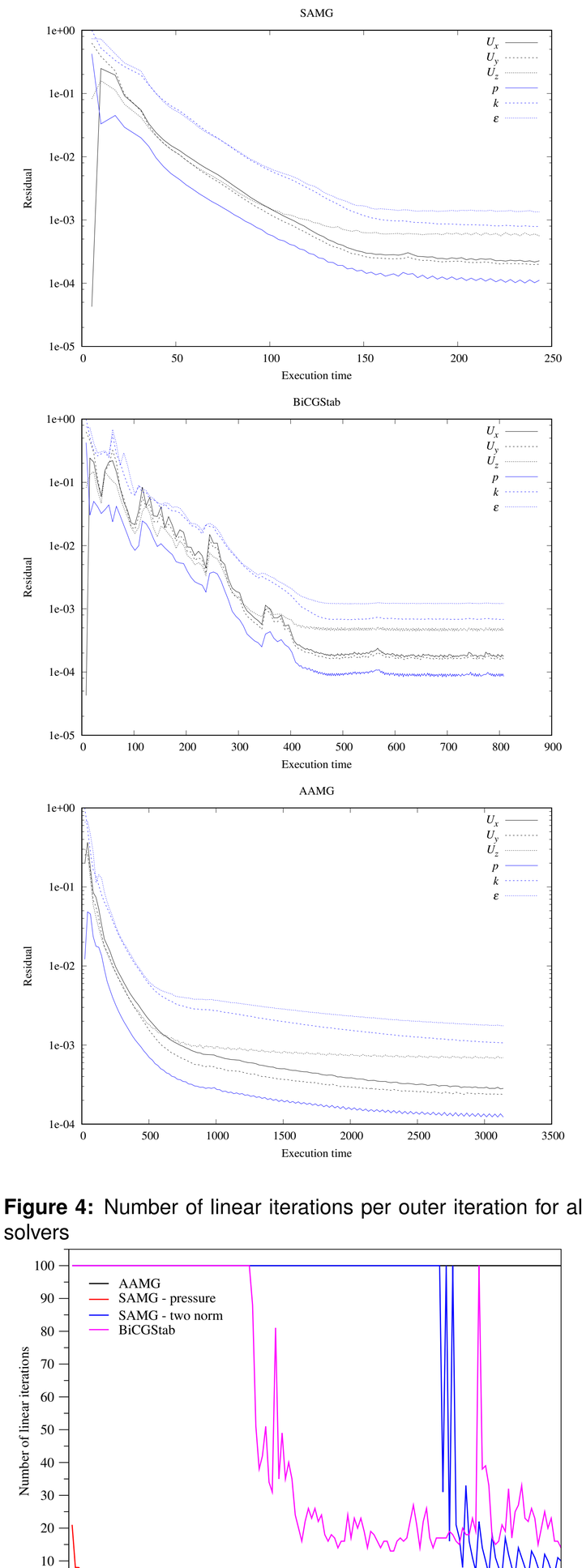

Figure 4: Number of linear iterations per outer iteration for all solvers

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